A Relativistic Look
at Quantum Field
Theory

Oxford
May 1986

CLASSICAL CONCEPT OF FIELD

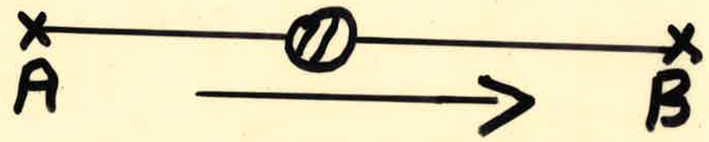
FIELD THEORY: Associates contain properties with Space-Time Points

Compare

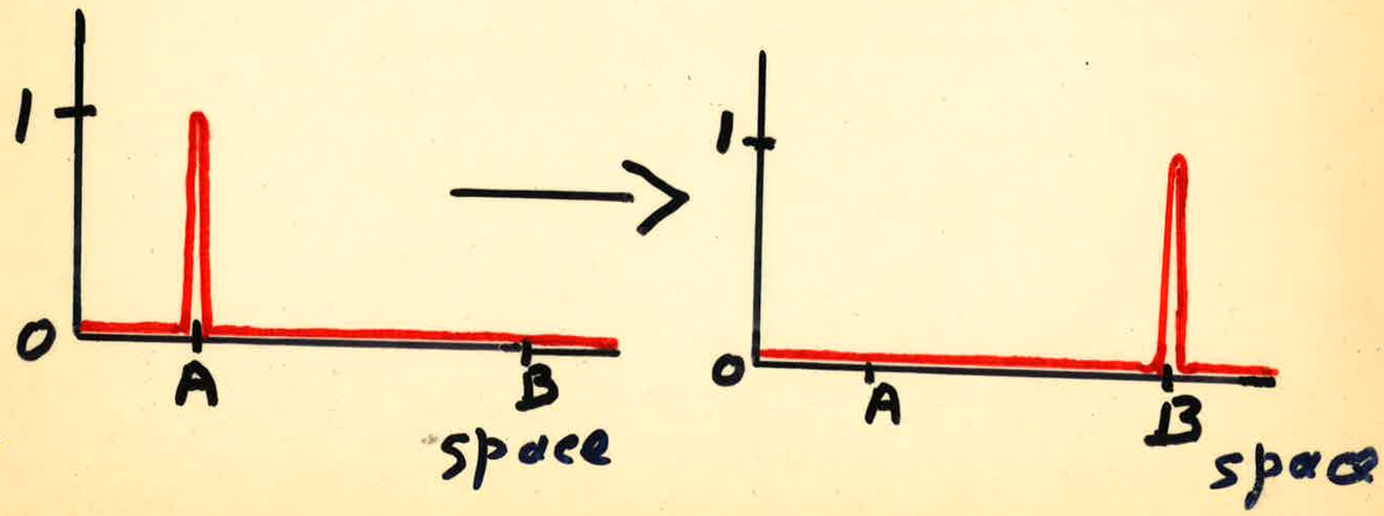
PARTICLE THEORY: Attributes to contain individuals (the particles) a variety of properties

MOTION FROM A TO B

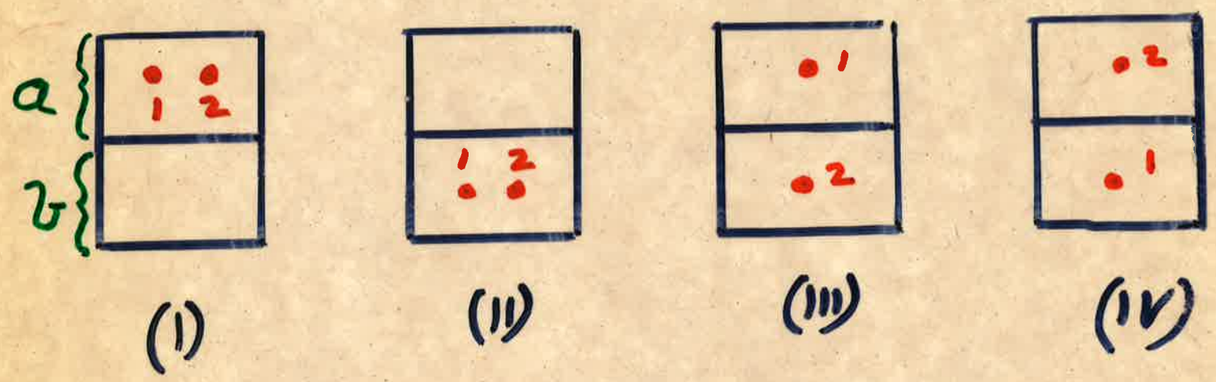
Particle Description



Field Description



Statistical weights for 2-particle System



(I) (II) (III) (IV)

In Quantum Stat. Mech. (III) and (IV) are regarded as one and the same state for the purposes of assigning Statistical weights

Quantum Statistical Mechanics

Consider the 4 product wave functions

$$\psi_a(\underline{r}_1)\cdot\psi_a(\underline{r}_2) \quad \text{I}$$

$$\psi_b(\underline{r}_1)\cdot\psi_b(\underline{r}_2 \quad \text{II}$$

$$\psi_a(\underline{r}_1)\cdot\psi_b(\underline{r}_2) \quad \text{III}$$

$$\psi_a(\underline{r}_2)\cdot\psi_b(\underline{r}_1) \quad \text{IV}$$

4-dimensional vector space equally well spanned by

Symmetric

$$\psi_a(\underline{r}_1)\cdot\psi_a(\underline{r}_2) \quad \text{V}$$

$$\psi_b(\underline{r}_1)\cdot\psi_b(\underline{r}_2) \quad \text{VI}$$

$$1/\sqrt{2}\,(\psi_a(\underline{r}_1)\cdot\psi_b(\underline{r}_2)+\psi_a(\underline{r}_2)\cdot\psi_b(\underline{r}_1)) \quad \text{VII}$$

Antisymmetric

$$1/\sqrt{2}\,(\psi_a(\underline{r}_1)\cdot\psi_b(\underline{r}_2)-\psi_a(\underline{r}_2)\cdot\psi_b(\underline{r}_1)) \quad \text{VIII}$$

THE INDISTINGUISHABILITY PRINCIPLE (IP)

Two particles are indistinguishable if $\langle P\phi|Q|P\phi\rangle = \langle\phi|Q|\phi\rangle$

$\forall Q, P, \phi$

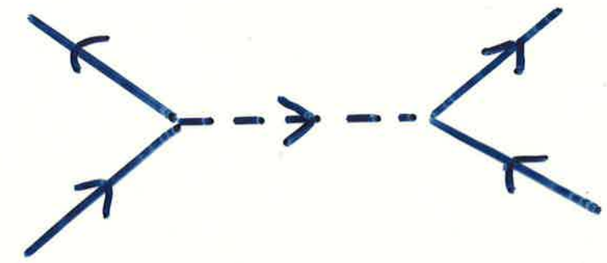
IP can be taken as a restriction on observables $\Rightarrow$ P commutes with Q, i.e. Q is a symmetric function of particle labels $\rightarrow$ parastatistics

or IP can be regarded as a restriction on states $\Rightarrow$ $P|\phi\rangle = \pm|\phi\rangle$

So Bosons and Fermions only allowed

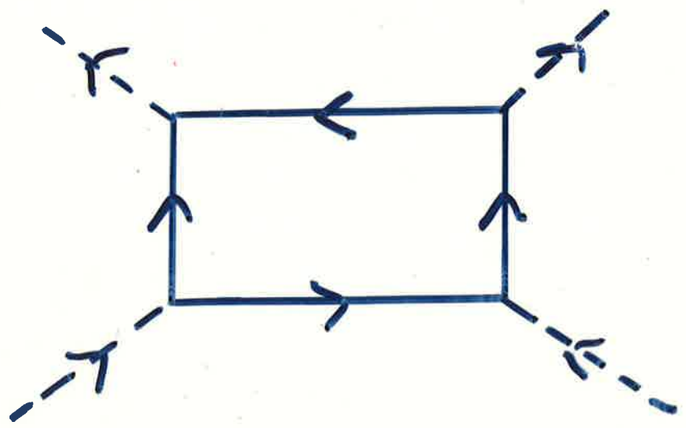
MATTER AND FORCE

Compare



---- photon
—— electron

with



So which is the 'matter' particle and which is the 'force' particle?

A Philosopher Looks at QFT Contd.

But 2nd quantization is more general than the N-Particle Schrödinger Eq. because of the constraint

$$\sum_i n_i = N$$

FOCK SPACE

$$\mathcal{F} = K_0 \oplus K_1 \oplus \cdots K_N \oplus \cdots$$

$\downarrow$ vacuum

CREATION and ANNIHILATION OPERATORS

$$\left.\begin{aligned} a_i \,|n_i\rangle &= \sqrt{n_i}\cdot|n_i-1\rangle \\ a_i^+ \,|n_i\rangle &= \sqrt{n_i+1}\cdot|n_i+1\rangle \end{aligned}\right\}$$